

Decoherence dynamics of open qubit systems

By

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Abstract

This thesis is contributed to the study of decoherence dynamics of the dissipative qubit system. We mainly concentrate on the profound impact of the formation of a bound state between the qubit and its local environment on the decoherence behavior of the reduced qubit system under the non-Markovian dynamics.

Firstly, we evaluate exactly the non-Markovian effect on the decoherence dynamics of a single qubit interacting with a dissipative vacuum reservoir. We find that the quantum coherence of the qubit can be partially trapped in the steady state when the non-Markovian memory effect of the reservoir is taken into account. Our analysis shows that it is the formation of a bound state between the qubit and its reservoir that results in this residual coherence in the steady state under the non-Markovian dynamics. A physical condition for the formation of the bound state is given explicitly. Our results suggest a potential way to decoherence control by modifying the system-reservoir interaction and the spectrum of the reservoir to the non-Markovian regime in the scenario of reservoir engineering.

Secondly, We study the entanglement dynamics of two qubits locally interacting with their reservoirs and explore the entanglement preservation under the non-Markovian dynamics. We show that the existence of a bound state of the qubit and its reservoir and the non-Markovian effect are two essential ingredients and their interplay plays a crucial role to preserve the entanglement in the steady state. When the non-Markovian effect is neglected, the entanglement sudden death is reproduced. On the other hand, when the non-Markovian is significantly strong but the bound state is absent, the phenomenon of the entanglement sudden death and its revival is recovered. Our formulation presents for the first time a unified picture about the entanglement preservation and provides a clear clue on how to preserve the entanglement in quantum information processing.

Finally, in order to obtain a thorough understanding of the entanglement dynamics, we study the entanglement distribution of a two-qubit system, each of which is embedded into its local reservoir, among all the bipartite subsystems including qubit-qubit, qubit-reservoir, and reservoir-reservoir. Different to the result that the entanglement of the qubits is transferred entirely to the reservoirs under the Markovian dynamics, we find that the entanglement can be stably distributed among all components under the

non-Markovian dynamics, and particularly it also satisfies an identity firstly given by Yöncü, Yu and Eberly [J. Phys. B **40**, S45 (2007)] for a double J-C model without decoherence. While the explicit distribution of the entanglement is dependent of the detail of the model, even the approximation used, the identity remains unchanged. Our unified treatment includes the previous results in the literature as special cases. The result reveals the profound nature of the entanglement and should have significant implications for quantum information processing.

This thesis may give a clear clue of decoherence dynamics under different approximations and how to preserve quantum coherence in the steady state.

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Chapter 1

Introduction

The superposition rule of quantum state, one of the fundamental principles of quantum mechanics, allows a quantum system to be in a linear and coherent superposition of all possible quantum states [1]. It leads to the quantum coherence, which is the essential difference of quantum world to the classical one. Quantum coherence is of great importance not only in understanding the basic rules of quantum mechanics but also in quantum information science [2].

Entanglement (also named as quantum correlation), as a non-local quantum coherence, is one of the characteristic trait of quantum mechanics [1]. A state in multipartite system is entangled when it cannot be written as the summation of the product states of the subsystems. On the one hand, entanglement relates to a lot of fundamental problems in quantum mechanics, such as, reality, local realism, hidden variables, and quantum measurement theory [3, 4, 5]. On the other hand, entanglement can be used as a kind of information resource to realize some missions of quantum information processing which are intractable for classical one, such as quantum communication [6, 7, 8], quantum computation [9, 10], and quantum cryptography [11].

Any realistic quantum system inevitably interacts with its surrounding environment which leads to the loss of quantum coherence, or decoherence of the quantum system. This ubiquitous phenomenon deteriorates the superposition and the entanglement of quantum state. In terms of information or energy, decoherence means that the information or energy flows from quan-

tum system to the environment irreversibly. The decoherence is deemed as one of the main obstacles to the realization of quantum information processing. Recently, much attention has been paid on the study of the dynamics of open quantum system, by which people want to get a thorough understanding to the detrimental effect caused by decoherence on quantum information processing and some clues on how to suppress this unwanted effect. In this thesis, we will concentrate on a detailed study of the decoherence dynamics of qubits influenced by their vacuum reservoirs under different environments or approximations and explore the potential dynamical suppression mechanism to the decoherence.

The dynamics of open quantum system is rather complicated because of the complex structure of the environment with which the system of interest interacts. Actually, the exactly solvable models are very few, only including the quantum Brownian motion and the system of a two-level atom in a vacuum reservoir with Lorentzian spectrum density [1]. Many approximations are ordinarily performed. A generally used approximation in the conventional investigation to the dynamics of open quantum system is the Born-Markovian approximation [12], which treats the interaction between the quantum system of interest and its environment perturbatively and neglects the memory effect of the environment. This approximation is valid when the coupling between quantum system and its environment is weak (Born approximation) and the environmental correlation time is small compared to the typical time scale of the quantum system (Markovian approximation). This approximation yields equations of motion such as Redfield or master equation, which is local in time and mathematically tractable, for the quantum system of interest. Based on this approximation, it is widely accepted that the quantum coherence of a single quantum system flows irreversibly to the environment and the decoherence dynamics can be simply depicted as an exponential decay. However, things are changed dramatically when entanglement dynamics which involves more subsystems, such as qubits, in the quantum system, are studied. some works have showed that the entanglement between two qubits ceases abruptly in a finite time scale [13, 14]. This remarkable phenomenon that the entanglement of the qubits under the

Markovian decoherence dynamics can be terminated in a finite time despite the coherence of single qubit lossing in an asymptotical manner is named as entanglement sudden death (ESD). Further investigation shows that such ESD is strongly related to the initial portion of double excitation component [15]. The larger the initial portion of the component is, the shorter the death time is. On the other hand if the environments are composed of the thermal or squeezed reservoirs, it is found that the ESD would always happen for any initial entangled state. Experimentally, the ESD has been observed using an all optical setup and atomic ensemble system [16, 17].

The Born-Markovian approximation simplifies greatly the mathematics to solve the dynamics of open quantum system, but it suffers more and more challenges under the newly emerging experimental results [18, 19, 20, 21, 22]. Especially, when the environment has certain structures, such as atom in cavity or photonic band gap (PBG) mediums [23, 24, 25, 26, 27, 28], the non-Markovian effect can not be neglected anymore. The non-Markovian effect is a kind of dynamic feedback effect which arises from the memory effect of environment. In terms of information or energy, the non-Markovian effect means that the information or energy flows back from the environment to the quantum system of interest. The study of open quantum dynamics beyond the Markovian approximation is rather complicated, which needs the solving of coupled integro-differential equations. The decoherence dynamics of quantum system in this case exhibits a dramatic deviation from the exponential decay behavior. Actually any kind of environment should have memory effect. When this memory effect is very weak, the Markovian approximation is applicable. On the other hand, when the memory effect of the environment is extremely strong, it would partially feed the lost coherence back to the quantum system. In this case the Markovian approximation would be not applicable. As far as the entanglement in two-qubit system is concerned, the non-Markovian effect also has a great impact on it. Modeling the environments as vacuum lossy cavities, Bellomo *et al.* showed that the entanglement would revive again after a finite period time of completely disappearance [29]. This is a solvable model and the entanglement dynamics can be analytically expressed. Via tuning relevant parameters, one can

easily observe that the non-Markovian effect postpones greatly the death of the qubits entanglement. Entanglement dynamics of continuous variable system has also been well studied and the non-Markovian effect also makes the entanglement dynamics oscillate [30, 31, 32, 33, 34], which can be understood as the backaction effect of the environment on the quantum system. The ESD and its revival due to the non-Markovian effect has been experimentally observed [22]. All these experimental and theoretical works show clearly that the coherence or entanglement time of the quantum system can be much enhanced by the non-Markovian effect.

However, in many cases such finite extension of the coherence/entanglement time is not enough for quantum information processing and thus it is desired to preserve a significant of the quantum coherence/entanglement, even partially, in the long time limit forever. Actually, some work has shown that it is realizable for some special environment cases. It has been found that the spontaneous emission of a two-level atom can be inhibited and its quantum coherence can be preserved when the atom is placed in a PBG material [26, 35, 36]. In the PBG material, the photonic mode density is zero within the PBG and this would be accompanied by the classical light localization and a photon-atom bound state. The excited-state population in this case is partially trapped, a phenomenon known as population trapping [27]. This result has been verified experimentally for quantum dot embedded in PBG material [37]. It has been realized that trapping the single-qubit population is the key step to protect entanglement in two-atom case [28]. When two initially entangled qubits are immersed in two separate PBG mediums, the entanglement of the two qubits can be preserved in the steady state with a large fraction. However the mechanism of entanglement preservation is still unclear. Also is this a general phenomenon in open quantum system or only available in this specific structured environment still is an open question.

To explore these questions, one should know the dynamics of quantum system not only in the short-time scale, but also in the long-time situation. In the short-time scale, when the non-Markovian effect is very strong, the quantum coherence would suffer transient oscillations manifesting the backaction effect of the memory environment. It is just the counteraction role played

by this backaction effect to the dissipation effect of the environment on the dynamics of the quantum system which results in the residual coherence of the quantum system in the long-time limit. Therefore, it is understandable that the non-Markovian effect is a prerequisite for the coherence preservation in the long-time limit. Then a natural question is: under the non-Markovian dynamics, what is the condition for the quantum coherence to be preserved in the long-time limit? This reminds us to examine the eigen solution of the whole system, which actually determines the long-time behaviors of quantum system. Firstly, let's consider the special case: the environment contains only one mode, which corresponds to the J-C model. The whole system possesses two real eigenvalues for each excitation-number subspace. Consequently, the quantum coherence of the two-level atom would experience loseless oscillations when the degree of freedom of the single-mode environment is traced out. However, when the environment possesses infinite modes, this would not be the truth anymore. It was shown that the real eigenvalues are not available anymore (except for the trivial ground state with eigenvalue being zero) and the complex eigenvalues are present when the environment has infinite modes [35, 38]. This is understandable based on the fact that the decay behavior of the quantum system under decoherence is just the effect taken by the imaginary part of the complex eigenvalues on the dynamics of reduced quantum system. However, John *et al* found that there is a real eigenvalue available when the environment is a PBG medium [35]. Physically, the existence of a real eigenvalue means the formation of atom-photon bound state. Due to the formation of bound state, spontaneous emission would be suppressed and a large proportion of quantum coherence may be preserved in the long-time limit. And Ref. [28] reported that the entanglement can be preserved with a large proportion in the long-time limit when two atoms are embedded in the PBG mediums. We argue that the suppression of the spontaneous emission and the entanglement preservation are both contributed from the formation of the bound state. Is this bound state available only for such PBG environment or for any environment? Under what condition the coherence or entanglement preservation is available for generic environments? These questions motivate us to do the investigation

in Chapters 2 and 3.

There is always lots of entanglement of the quantum system lost irrespective of the entanglement could be (partially) preserved or not. Then a nature question is: where does the lost entangle go? Modeling the whole systems as double J-C model, authors in Ref. [39] have shown that the entanglement oscillates between the atoms and the cavities in a lossless way. This is understandable since there is no decoherence in the J-C model. Via introducing a normalized collective state, authors in Ref. [40] showed that the initial entanglement between the qubits flows entirely to their local environments under the Markovian dynamics. We argue that things would be completely different under the non-Markovian decoherence dynamics. This judgement is based on the following observations. Firstly, it is possible to preserve some entanglement in the quantum system under the non-Markovian dynamics, which means that not all of the entanglement between the subsystems is transferred to their environments. Secondly, the entanglement preservation is due to the formation of the bound state between each subsystem and its environment. Therefore, entanglement would exist between each subsystem and its local environment. From these facts, we can see that the question where does the entanglement go should be reevaluated when the non-Markovian effect is taken into account. This motivates us to do the investigation in Chapter 4.

This thesis is organized as follows. In Chapter 2, the decoherence dynamics of single qubit is studied. We investigate the exact decoherence dynamics of a dissipative qubit coupling to a vacuum reservoir. We also study the static eigenvalue problem and give the condition when atom-reservoir bound state is formed, via which we reveal the mechanism of dynamical decoherence suppression due to the bound state. In Chapter 3, entanglement dynamics of two qubits under the influence of two independent vacuum reservoirs is studied. We give a mechanism of entanglement preservation. In Chapter 4, we study the entanglement distribution among all possible bipartite partitions of the same system. Finally, a summary of this thesis and the outlook of future works are given in Chapter 5.

Chapter 2

Decoherence dynamics of a dissipative qubit

In this chapter we study the exact decoherence dynamics of a single qubit (two-level atom) in a vacuum reservoir. We compare this result with the one obtained under the Markovian approximation. We also study the formation of bound state on the decoherence suppression.

To solve the dynamics for the general open quantum system is rather tricky. Here we consider that the environment, with which the qubit interacts, is in a vacuum state initially. Combining with numerical calculations, we can obtain the exact decoherence dynamics of the dissipative qubit.

2.1 Introduction

Any realistic quantum system inevitably interacts with its surrounding environment, which leads to the loss of coherence, or decoherence, of the quantum system [1]. The decoherence of quantum bit (qubit) is deemed as a main obstacle to the realization of quantum computation and quantum information processing [2]. Understanding and suppressing the decoherence are therefore a major issue in quantum information science. For a Markovian environment, it is well known that the coherence of a qubit experiences an exponential decrease [1]. To beat this unwanted degradation, many controlling strategies, passive or active, have been proposed [41, 42, 43, 44, 45].

In recent years much attention has been paid to the non-Markovian effect

on the decoherence dynamics of open quantum system [46, 47, 48, 49, 50, 51, 52, 53]. The significance of the non-Markovian dynamics in the study of open quantum system is twofold. i) It is of fundamental interest to extend the well-developed methods and concepts of Markovian dynamics to non-Markovian case [1, 12] for the open quantum system in its own right. ii) There are many new physical situations in which the Markovian assumption usually used is not fulfilled and thus the non-Markovian dynamics has to be introduced. In particular, many experimental results have evidenced the existence of the non-Markovian effect [18, 19, 20, 21], which indicates that one can now approach the non-Markovian regime via tuning the relevant parameters of the system and the reservoir. The non-Markovian effect means that the environment, when its state is changed due to the interaction with the quantum system, in turn, exerts its dynamical influence back on the system. Consequently one can expect decoherence dynamics of the quantum system could exhibit a dramatic deviation from the exponential decaying behavior. In 2005, DiVincenzo and Loss studied the decoherence dynamics of the spin-boson model for the Ohmic heat bath in the weak-coupling limit. They used the Born approximation and found that the coherence dynamics has a power-law behavior at long-time scale [54], which greatly prolongs the coherence time of the quantum system. Such power-law behavior suggests that the non-Markovian effect may play a constructive role in suppressing decoherence of the system. Nevertheless, in many cases the finite extension of the coherence time of the system is not sufficient for the quantum information processing, a question arises whether the coherence of the system can be preserved in the long-time limit, even partially. Theoretically, the answer is positive if the environment has a nontrivial structure. It has been shown that some residual coherence can be preserved in the long-time steady state when the environment is a periodic band gap material [25, 26, 27, 28] or leaky cavity [23, 24]. It is stressed that the residual coherence is due to the confined structured environment. A natural question is: Whether the coherence of the system can be dynamically preserved or not by the non-Markovian effect if the environment has no any special structure, e.g., a vacuum reservoir?

In this chapter, we study the exact decoherence dynamics of a qubit in-

interacting with a vacuum reservoir and examine the possibility of decoherence suppression using the non-Markovian effect. The main aim of this chapter is to analyze if and how the coherence present in the initial state can be trapped with a noticeable fraction in the steady state even when the environment is consisted of a vacuum reservoir with trivial structure. We show that the non-Markovian effect manifests its action on the qubit not only in the transient dynamical process, but also in the asymptotical behavior. Our analysis shows that the physical mechanism behind this dynamical suppression to decoherence is the formation of a bound state between the qubit and the reservoir. The no-decaying character of the bound state leads to the inhibition of the decoherence and the residual coherence trapped in the steady state. A similar vacuum induced coherence trapping in the continuous variable system has been reported in [33, 34]. Such coherence trapping phenomenon provides an alternative way to suppress decoherence. This could be realized by controlling and modifying the system-reservoir interaction and the properties of the reservoir [25] by the recently developed reservoir engineering technique [55, 56, 57].

2.2 The model and exact decoherence dynamics of the qubit

We consider a qubit (two-level atom) which interacts with a vacuum quantized radiation electromagnetic field. The Hamiltonian of the total system reads [1]

$$H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (g_k \sigma_+ a_k + h.c.), \quad (2.1)$$

where ω_0 is the transition frequency and σ_\pm is the raising and lowering operators of the qubit and a_k^\dagger and a_k , respectively, are the creation and annihilation operators of the \mathbf{k} -th mode with frequency ω_k of the radiation field. The coupling strength between the qubit and the radiation field is given by

$$g_k = -i \sqrt{\frac{\omega_k}{2\varepsilon_0 V}} \hat{\mathbf{e}}_k \cdot \mathbf{d}, \quad (2.2)$$

where $\hat{\mathbf{e}}_k$ and \mathbf{d} are unit polarization vector of the radiation field and the dipole moment of the qubit respectively. Throughout this paper we assume

$\hbar = 1$.

To obtain the exact dynamics of the qubit, we first consider the following two simple cases. For simplicity, we assume there is no correlation between the qubit and its reservoir at the initial time $t = 0$. If the initial state of the system is $|\Psi(0)\rangle = |-, \{0_k\}\rangle$, where $|-\rangle$ denotes the ground state of the qubit and $|\{0_k\}\rangle$ represents the vacuum state of the reservoir, the whole system will not evolve with time under the Hamiltonian (2.1). Whereas to the initial state $|\Psi(0)\rangle = |+, \{0_k\}\rangle$, in which $|+\rangle$ denotes the excited state of the qubit, the time evolution of the total system has the following form

$$|\Psi(t)\rangle = b_0(t) |+, \{0_k\}\rangle + \sum_k b_k(t) |-, \{1_k\}\rangle, \quad (2.3)$$

where $|\{1_k\}\rangle$ represents the field state containing one photon in the \mathbf{k} -th mode. Applying the Schrödinger equation, we get the time evolution of the probability amplitudes

$$i\dot{b}_0(t) = b_0(t)\omega_0 + \sum_k g_k b_k(t), \quad (2.4)$$

$$i\dot{b}_k(t) = b_k(t)\omega_k + g_k^* b_0(t), \quad (2.5)$$

where the superscript dot represents the differential with respect to time. Solving Eq. (2.5) formally and substituting the solution into Eq. (2.4), we can obtain

$$\dot{b}_0(t) + i\omega_0 b_0(t) = - \int_0^t f(t - \tau) b_0(\tau) d\tau, \quad (2.6)$$

where the kernel function is $f(x) = \sum_{k=0}^{\infty} |g_k|^2 \exp(-i\omega_k x)$. Obviously the memory effect has been registered in the kernel function. In the continuous limit of the environment frequency, the kernel function has the form

$$f(x) = \int_0^{\infty} J(\omega) e^{-i\omega x} d\omega, \quad (2.7)$$

where $J(\omega) = \eta \omega^3 e^{\frac{-\omega}{\omega_c}}$ is the spectral density [58], which characterizes the coupling strength of the reservoir to the qubit with respect to the reservoir frequency and $\eta = \frac{f|\hat{\mathbf{e}}_k \cdot \mathbf{d}|^2 d\Omega}{(2\pi c)^3 2\varepsilon_0}$. To eliminate infinity in frequency integration, we have introduced the cutoff frequency ω_c . On physical grounds, the introducing of the cutoff frequency means that not all of the infinite modes of the

reservoir contribute to the interaction with the qubit, and one always expects the spectral density going to zero for the modes with frequencies higher than certain characteristic frequency. It is just this characteristic frequency which determines the specific behavior and the properties of the reservoir. One can see that in our model, the spectral density has a super-Ohmic form [58].

From the time evolution of the above two situations, one can get the time evolution of any given initial state of the system readily. For an initially mixed state, which is described by the following density operator

$$\begin{aligned} \rho_{\text{tot}}(0) = & (\rho_{11} |+\rangle \langle +| + \rho_{12} |+\rangle \langle -| + \rho_{21} |-\rangle \langle +| \\ & + \rho_{22} |-\rangle \langle -|) \otimes |\{0\}_k\rangle \langle \{0\}_k|. \end{aligned} \quad (2.8)$$

The time evolution of the total system can be calculated explicitly. In fact, what we care about is the reduced density matrix of the qubit, which is obtained by tracing over the reservoir variables

$$\rho(t) = \begin{pmatrix} \rho_{11} |b_0(t)|^2 & \rho_{12} b_0(t) \\ \rho_{21} b_0^*(t) & 1 - \rho_{11} |b_0(t)|^2 \end{pmatrix}. \quad (2.9)$$

Differentiating Eq. (2.9) with respect to time, we may obtain the equation of motion of the qubit

$$\begin{aligned} \dot{\rho}(t) = & -i \frac{\Omega(t)}{2} [\sigma_+ \sigma_-, \rho(t)] + \frac{\gamma(t)}{2} [2\sigma_- \rho(t) \sigma_+ \\ & - \sigma_+ \sigma_- \rho(t) - \rho(t) \sigma_+ \sigma_-], \end{aligned} \quad (2.10)$$

where $\Omega(t) = -2\text{Im}[\frac{\dot{b}_0(t)}{b_0(t)}]$ and $\gamma(t) = -2\text{Re}[\frac{\dot{b}_0(t)}{b_0(t)}]$. $\Omega(t)$ plays the role of time-dependent shifted frequency and $\gamma(t)$ that of time-dependent decay rate [1]. It is worth mentioning that during the derivation of master equation (2.10) we have not resorted to the Born-Markovian approximation. Therefore Eq. (2.10) is the exact master equation of the qubit system.

It is interesting to notice that one can reproduce the conventional Markovian one from our exact non-Markovian master equation under certain approximations. By redefining the probability amplitude as $b_0(\tau) = b'_0(\tau)e^{-i\omega_0\tau}$, one can recast Eq. (2.6) into

$$\dot{b}'_0(t) + \int_0^\infty d\omega J(\omega) \int_0^t d\tau e^{i(\omega_0 - \omega)(t - \tau)} b'_0(\tau) = 0, \quad (2.11)$$

where $J(\omega)$ is defined the same as above. Then, we take the Markovian approximation,

$$b'_0(\tau) \cong b'_0(t), \quad (2.12)$$

namely, approximately taking the dynamical variable to the one that depend only on the present time so that any memory regarding the earlier time is ignored. The Markovian approximation is mainly based on the physical assumption that the correlation time of the reservoir is very small compared with the typical time scale of system evolution. Also under this assumption we can extend the upper limit of the τ integration in Eqs. (2.11) to infinity and use the equality

$$\lim_{t \rightarrow \infty} \int_0^t d\tau e^{\pm i(\omega_0 - \omega)(t - \tau)} = \pi \delta(\omega - \omega_0) \mp iP \left(\frac{1}{\omega - \omega_0} \right), \quad (2.13)$$

where P and the delta-function denote the Cauchy principal value and the singularity, respectively. The integro-differential equation in (2.11) is thus reduced to a linear ordinary differential equation. The solutions of b'_0 as well as b_0 can then be easily obtained as

$$b_0(t) = e^{-i(\omega_0 - \delta\omega)t - \pi J(\omega_0)t}, \quad (2.14)$$

where $\delta\omega = P \int_0^\infty \frac{J(\omega)d\omega}{\omega - \omega_0}$. Thus one can verify that,

$$\gamma(t) \equiv \gamma_0 = 2\pi J(\omega_0), \quad \Omega(t) \equiv \Omega_0 = 2(\omega_0 - \delta\omega), \quad (2.15)$$

which are exactly the coefficients in the Markovian master equation of the two-level atom system [1].

2.3 Purity and decoherence factor

To quantify the decoherence dynamics of the qubit, we introduce the following two quantities. The first one is the purity, which is defined as [2]

$$p(t) = \text{Tr} \rho^2(t). \quad (2.16)$$

Clearly $p = 1$ for pure state and $p < 1$ for mixed state. The second quantity describing the decoherence is the decoherence factor $c(t)$ of the qubit, which

is determined by the off-diagonal elements of the reduced density matrix

$$|\rho_{12}(t)| = c(t) |\rho_{12}(0)|. \quad (2.17)$$

The decoherence factor maintains unity when the reservoir is absent and vanishes for the case of completely decoherence.

For definiteness, we consider the following initial pure state of the qubit

$$|\psi(0)\rangle = \alpha |+\rangle + \beta |-\rangle, \quad (2.18)$$

in which α and β satisfy the normalization condition. Using Eq. (2.9), the exact time evolution of the qubit is easily obtained

$$\rho(t) = \begin{pmatrix} |\alpha|^2 |b_0(t)|^2 & \alpha\beta^* b_0(t) \\ \alpha^* \beta b_0^*(t) & 1 - |\alpha|^2 |b_0(t)|^2 \end{pmatrix}. \quad (2.19)$$

With Eq. (2.19), the purity and decoherence factor can be expressed explicitly

$$p(t) = 2 |\alpha|^4 |b_0(t)|^2 [|b_0(t)|^2 - 1] + 1, \quad (2.20)$$

and

$$c(t) = |b_0(t)|. \quad (2.21)$$

It is easy to verify, under the Born-Markovian approximation, the purity and decoherence factor have the following forms

$$p(t) = 2 |\alpha|^4 e^{-\gamma_0 t} (e^{-\gamma_0 t} - 1) + 1, \quad (2.22)$$

and

$$c(t) = e^{-\frac{\gamma_0}{2} t}, \quad (2.23)$$

where the time-independent decay rate γ_0 is given in Eq. (2.15). Obviously, the system asymptotically loses its quantum coherence ($c(\infty) = 0$) and approaches a pure steady state ($p(\infty) = 1$) irrespective of the form of the initial state under the Markovian approximation. One can also find from Eqs. (2.20-2.23) that the probability amplitude of excited state plays key role in the decoherence dynamics.

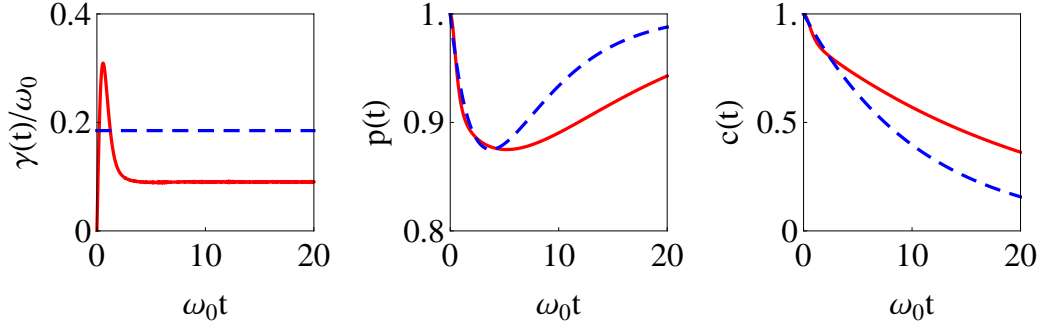


Figure 2.1: Time evolution of $\gamma(t)$, $p(t)$ and $c(t)$ in non-Markovian situation (solid line) and the corresponding Markovian situation (dashed line), when η and ω_c are small. The parameters used here are $\alpha = 1/\sqrt{2}$, $\eta = 0.08$ and $\omega_c/\omega_0 = 1.0$.

2.4 Numerical results and analysis

In this section, by numerically solving Eq. (2.6), we study the influence of memory effect of reservoir on the exact dynamics of the qubit. Noticing the fact that the memory effect registered in the kernel function is essentially determined by the spectrum density $J(\omega)$, one can expect that $J(\omega)$ plays an major role in the exact dynamics of the qubit. In the following, we show how the decoherence of the qubit can be fully suppressed under the non-Markovian dynamics in terms of the relevant parameters of $J(\omega)$ [59].

2.4.1 The influence of coupling constant

In the following, we numerically analyze the exact decoherence dynamics of the qubit with respect to decay rate $\gamma(t)$, purity $p(t)$ and decoherence factor $c(t)$ in terms of the coupling constant η [59].

In Fig. 2.1 we plot the time evolution of decay rate $\gamma(t)$, purity $p(t)$, decoherence factor $c(t)$ and their Markovian correspondences in the weak coupling and low cutoff frequency case. We can see that $\gamma(t)$ shows distinct difference from its Markovian counterpart over a very short time interval. With time, $\gamma(t)$ tends to a definite positive value. The small “jolt” of $\gamma(t)$ in the short time interval just evidences the backaction of the memory effect of the reservoir exerted on the qubit [60]. It manifests that the reservoir

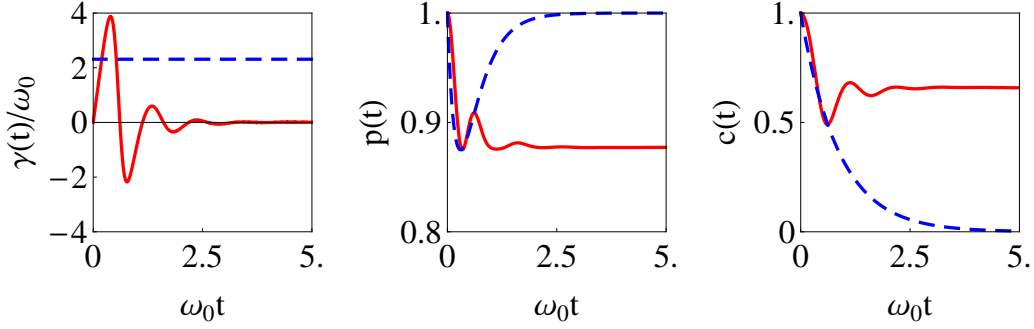


Figure 2.2: Time evolution of $\gamma(t)$, $p(t)$ and $c(t)$ in non-Markovian situation (solid line) and the corresponding Markovian situation (dashed line), when η is large. The parameters used here are $\alpha = 1/\sqrt{2}$, $\eta = 1.0$ and $\omega_c/\omega_0 = 1.0$.

does not exert decoherence on the qubit abruptly, just as the result based on Markovian approximation, but dynamically influences the qubit and gradually establishes a stable decay rate to the qubit. Furthermore, it is also shown that the decay rate is positive in the full range of evolution, which results in any initial qubit state evolving to the ground state $|\psi(\infty)\rangle = |-\rangle$ irreversibly. Consequently the decoherence factor monotonously decreases to zero with time and the purity approaches unity in the long-time limit, which is consistent with the result under Markovian approximation. The result indicates that although the reservoir has backaction effect on the qubit, it is quite small. And the dissipation effect of the reservoir dominates the dynamics of the qubit. Thus no qualitative difference can be expected between the exact result and the Markovian one with the backaction effect ignored. Therefore the widely used Markovian approximation is applicable in this case. Nevertheless, at the short and immediate time scales the overall behavior is still quite different from that of the Markovian dynamics. The decoherence factor shown in the right-hand panel of Fig. 2.1 shows non-exponential decay, which is in agreement with the result obtained previously in the spin-boson model in the weak-coupling limit [54]. However, the situation is dramatically changed if the coupling is strengthened as discussed below.

With the same cutoff frequency as in Fig. 2.1 but a larger coupling constant, we plot in Fig. 2.2 the decay rate, purity and decoherence factor

in the strong coupling case. In this case the non-negligible backaction of the reservoir has a great impact on the dynamics of the qubit. Firstly, we can see that the decay rate not only exhibits oscillations, but also takes negative values in the short time scale. Physically, the negative decay rate is a sign of strong backaction induced by the non-Markovian memory effect of the reservoir. And the oscillations of the decay rate between negative and positive values reflect the exchange of excitation back and forth between qubit and the reservoir [48]. Consequently both the decoherence factor and the purity exhibit oscillations in a short-time scale, which shows dramatic deviation to the Markovian result. Therefore, entirely different to the weak coupling case in Fig. 2.1, the reservoir in the strong coupling case here has strong backaction effect on the qubit. Secondly, we also notice that the decay rate approaches zero in the long-time limit. The vanishing decay rate means, after several rounds of oscillation, the qubit ceases decaying asymptotically. The non-Markovian purity maintains a steady value asymptotically, which is less than unity. This indicates that the steady state of the qubit is not the ground state anymore, but a mixed state. The decoherence factor also tends to a non-zero value, which implies that the coherence of the qubit is preserved with a noticeable fraction in the long-time steady state. These phenomena, which are qualitatively different to the Markovian situation, manifest that the memory effect has a considerable contribution not only to the short-time, but also to the long-time behavior of the decoherence dynamics. The presence of the residual coherence in the steady state also suggests a potential active control way to protect quantum coherence of the qubit from decoherence via the non-Markovian effect.

2.4.2 The influence of cutoff frequency

The cutoff frequency ω_c , on the one hand, is introduced to eliminate the infinity in the frequency integration. On the other hand it also determines the frequency range in which the power form is valid [61]. In the following, we elucidate the influence of cutoff frequency on the exact decoherence dynamics [59].

Fixing η as the value in Fig. 2.1 and increasing the cutoff frequency, we

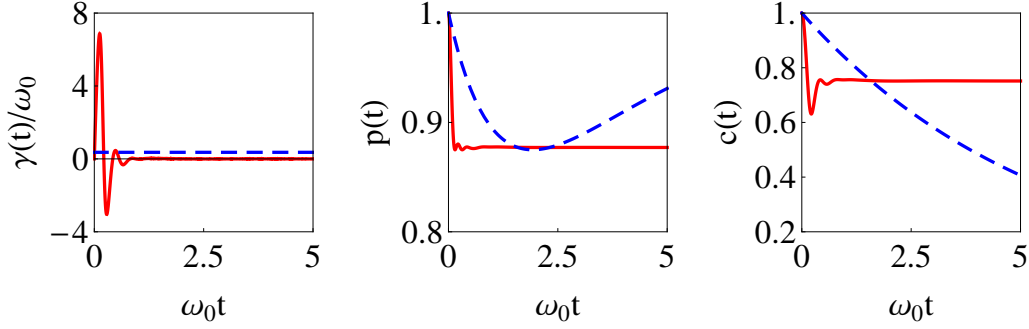


Figure 2.3: Time evolution of $\gamma(t)$, $p(t)$ and $c(t)$ in non-Markovian situation (solid line) and the corresponding Markovian situation (dashed line), when ω_c is large. The parameters used here are $\alpha = 1/\sqrt{2}$, $\eta = 0.08$ and $\omega_c/\omega_0 = 3.0$.

plot in Fig. 2.3 the dynamics of the qubit in a high cutoff frequency case. It shows that a similar decoherence behavior as the strong coupling case in Fig. 2.2 can be obtained. After several rounds of oscillation, the decay rate tends to zero in the long-time limit. The negative decay rate makes the lost coherence partially recovered. The vanishing decay rate in the long-time limit results in the decoherence frozen before the qubit gets to its ground state. Thus there is some residual coherence trapped in the steady state. Similar to the strong coupling case, it is essentially the interplay between the backaction and the dissipation on the dynamics of qubit which results in the inhibition of decoherence. We argue that in this high cutoff frequency regime, the widely used Markovian approximation is not applicable because of the strong backaction effect of the reservoir.

2.4.3 The physical mechanism of the decoherence inhibition: the formation of atom-photon bound state

From the analysis above we can see clearly that the decoherence can be inhibited in the non-Markovian dynamics. A natural question is: What is physical mechanism to cause such dynamical decoherence inhibition? To answer this question, let us find the eigen solution of Eq. (2.1) in the sector of one-excitation in which we are interested [59]. The eigenequation reads

$H|\varphi_E\rangle = E|\varphi_E\rangle$, where $|\varphi_E\rangle = c_0|+, \{0_k\}\rangle + \sum_{k=0}^{\infty} c_k|-, 1_k\rangle$. After some algebraic calculation, we can obtain a transcendental equation of E

$$y(E) \equiv \omega_0 - \int_0^{\infty} \frac{J(\omega)}{\omega - E} d\omega = E. \quad (2.24)$$

From the fact that $y(E)$ decreases monotonically with the increase of E when $E < 0$ we can say that if the condition $y(0) < 0$, i.e.

$$\omega_0 - 2\eta \frac{\omega_c^3}{\omega_0^2} < 0 \quad (2.25)$$

is satisfied, $y(E)$ always has one and only one intersection in the regime $E < 0$ with the function on the right-hand side of Eq. (2.24). Then the system will have an eigenstate with real (negative) eigenvalue, which is a bound state [38, 59], in the Hilbert space of the qubit plus its reservoir. While in the regime of $E > 0$, one can see that $y(E)$ is divergent, which means that no real root E can make Eq. (2.24) well-defined. Consequently Eq. (2.24) does not have positive real root to support the existence of a further bound state. It is noted that Eq. (2.24) may possess complex root. Physically this means that the corresponding eigenstate experiences decay contributed from the imaginary part of the eigenvalue during the time evolution, which causes the excited-state population approaching zero asymptotically and the decoherence of the reduced qubit system.

The formation of bound state is just the physical mechanism responsible for the inhibition of decoherence. This is because a bound state is actually a stationary state with a vanishing decay rate during the time evolution. Thus the population probability of the atomic excited state in bound state is constant in time, which is named as “population trapping” [25, 27]. This claim is fully verified by our numerical results. The parameters in Fig. 2.1 do not satisfy the condition (2.25) to support the existence of a bound state, then the dynamics experiences a severe decoherence. While with the increase of either η (in Fig. 2.2) or ω_c (in Fig. 2.3), the bound state is formed. Then the system and its environment is so correlated that it causes the decay rate of the system in the non-Markovian dynamics exhibiting: 1) transient negative value due to the backaction of the environment; 2) vanishing asymptotic value. Such interesting phenomenon, i.e. the vanishing asymptotical

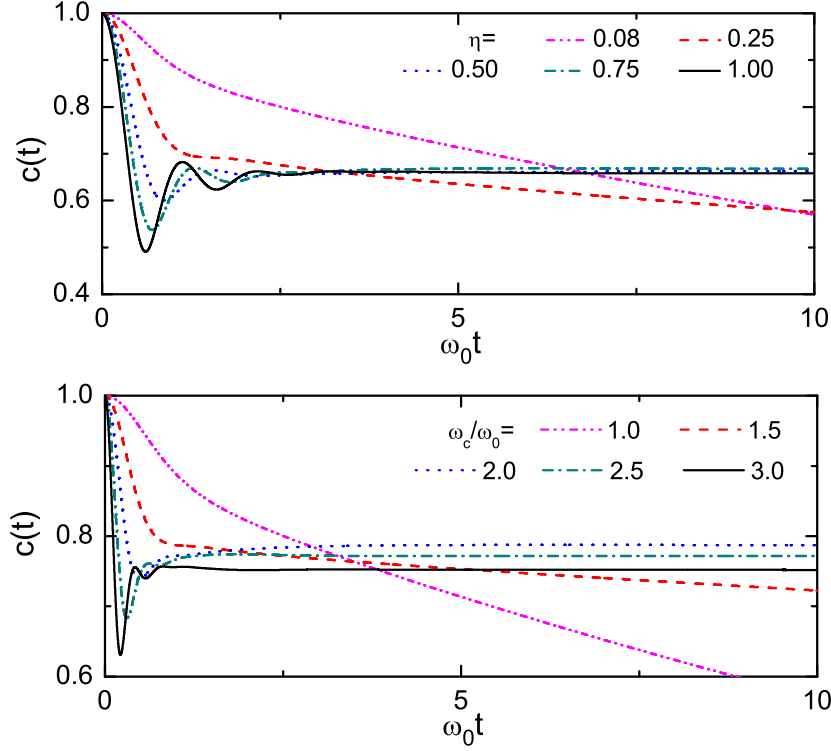


Figure 2.4: Time evolution of $c(t)$ in the non-Markovian dynamics with different η when $\omega_c/\omega_0 = 1.0$ (upper panel) and with different ω_c when $\eta = 0.08$ (lower panel).

decay rate in the large cutoff frequency regime for super-Ohmic spectrum density, was also revealed in Ref. [62]. This effect of course is missing in the conventional Born-Markovian decoherence theory, where the reservoir is memoryless.

In order to understand the exact decoherence dynamics more completely, we plot in Fig. 2.4 the crossover from coherence destroying to coherence trapping via increasing either the coupling constant or the cutoff frequency. Coherence trapping can be achieved as long as the bound state is formed. Therefore, one can preserve coherence via tuning the relevant parameters of system and the reservoir, e.g. the qubit-reservoir coupling constant and the property of the reservoir so that the condition (2.25) is satisfied.

2.5 Summary

In summary, we have investigated the exact decoherence dynamics of a qubit in a dissipative vacuum reservoir. We have found that even in a vacuum environment without any nontrivial structure, we can still get the decoherence suppression of the qubit owing to the dynamical mechanism of the non-Markovian effect. From our analytic and numerical results, we find that the non-Markovian reservoir has dual effects on the qubit: dissipation and backaction. The dissipation effect exhausts the coherence of the qubit, whereas the backaction one revives it. In the strong coupling and/or high cutoff frequency regimes, a bound state between the qubit and its reservoir is formed. It induces a strong backaction effect in the dynamics because the reservoir is strongly correlated with the qubit in the bound state. Furthermore, because of the non-decay character of the bound state the decay rate in this situation approach zero asymptotically. The vanishing of the decay rate causes the decoherence to cease before the qubit decays to its ground state. Thus the qubit in the non-Markovian dynamics would evolve to a non-ground steady state and there is some residual coherence preserved in the long-time limit. Our results make it clear how the non-Markovian effect shows its effects on the decoherence dynamics in different parameter regimes.

The presence of such coherence trapping phenomenon actually gives us an active way to suppress decoherence via non-Markovian effect. This could be achieved by modifying the properties of the reservoir to approach the non-Markovian regime via the potential usage of the reservoir engineering technique [55, 56, 57, 63]. Many experimental platforms, e.g. mesoscopic ion trap [55, 56], cold atom BEC [57], and the photonic crystal material [25] have exhibited the controllability of decoherence behavior of relevant quantum system via well designing the size (i.e. modifying the spectrum) of the reservoir and/or the coupling strength between the system and the reservoir. It is also worth mentioning that a proposal aimed at simulating the spin-Boson model, which is relevant to the one considered in this paper, has been reported in the trapped ion system [64]. On the other side many practical systems can now be engineered to show the novel non-Markovian

effect [19, 20, 21, 22]. All these achievements show that the recent advances have paved the way to experimentally simulate the paradigmatic models of open quantum system, which is one part of the new-emergent field, quantum simulators [65]. Our work sheds new light on the way to indirectly control and manipulate the dynamics of quantum system in this experimental platforms.

A final remark is that our results can be generalized to the system consisted of two qubits, each of which interacts with a local reservoir. Because of the coherence trapping we expect that the non-Markovian effect plays constructive role in the entanglement preservation [28, 34, 66].

Chapter 3

Mechanism of entanglement preservation

In this chapter, we study the mechanism of entanglement preservation. We found entanglement can be preserved in the long time limit as long as bound states are formed in the local systems. We also find the non-Markovian effect has a profound effect on the entanglement preservation.

3.1 Introduction

Entanglement is not only of fundamental interest to quantum mechanics, but also of great importance to quantum information processing [2]. However, due to the inevitable interaction of qubits with their environments, entanglement always experiences degradation. A phenomenon that the entanglement between two qubits may completely disappear at a finite time, known as “entanglement sudden death” (ESD), has been predicted theoretically [13, 14] and subsequently been verified experimentally [16, 17], which indicates the specific behavior of the entanglement different from the coherence. From the point of view of applications, the ESD is apparently disadvantageous to the quantum information processing.

Recently, Bellomo *et al.* [29] found that the entanglement can revive after some time interval of the ESD and thus extends significantly the entangled time of the qubits. This remarkable phenomenon, which has been experimentally observed [22], is physically due to the dynamical backaction, i.e., the

non-Markovian effect, of the memory environments. However, in many cases the finite extension of the entangled time is not enough and thus it is desired to preserve a significant fraction of the entanglement in the long time limit. Indeed, it was shown [28] that some noticeable fraction of entanglement can be obtained by engineering structured environment such as photonic band-gap materials [35]. According to these works, it is still unclear if the residual entanglement is fundamentally due to the specific structured materials or due to certain physical mechanism. Is there any essential relationship between the ESD and/or its revival phenomena and the residual entanglement?

In this chapter we focus on these questions and elucidate the physical nature of the residual entanglement. Before proceeding, it is helpful to recall the physics of quantum electrodynamics of a single two-level atom placed in a dielectric with a photonic band gap [26, 35]. The coupling between the excited atom and electromagnetic vacuum in the dielectric leads to a novel photon-atom bound state, in which the fractional atomic population on the excited state occurs, also known as population trapping [27]. This result has been verified experimentally for quantum dots embedded in photonic band-gap environment [37]. The population trapping has been directly connected to the entanglement trapping due to the structured environment [28]. Here we reveal for the first time that there are two essential conditions to preserve the entanglement. One is the existence of the bound state between the system and its environment, which provides an ability to preserve the entanglement, and the other is the non-Markovian effect, which provides a way to preserve the entanglement. Our result can reproduce the ESD [14] when the non-Markovian effect is neglected. The phenomenon of the ESD and its revival discussed in Ref. [29] results from the non-Markovian effect when the bound state is not available. The interplay between the availability of the bound state and the non-Markovian effect can lead to a significant fraction of the entanglement preserved in the steady state. We verify these results by considering two reservoirs modeled by the super-Ohmic and Lorentzian spectra, respectively. The result provides a general method on how to protect the entanglement by engineering the environment.

3.2 The model and entanglement dynamics

We now consider two spatially separated systems A and B, each has a two-level atom coupled to a vacuum reservoir, and the two qubits are initially entangled but have no direct interaction. Owing to the independence of the two systems [29], we can investigate single “qubit + reservoir” system at the first place, then extend our studies to the double-one.

The single “qubit + reservoir” system can be formulated by the following Hamiltonian

$$H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k b_k^\dagger b_k + \sum_k (g_k \sigma_+ b_k + g_k^* \sigma_- b_k^\dagger), \quad (3.1)$$

where ω_0 is the transition frequency of the two-level atom, and σ_\pm are the atom raising and lowering operators, b_k^\dagger and b_k are respectively the creation and annihilation operators of the \mathbf{k} -th mode with frequency ω_k of the reservoir. g_k denotes the coupling strength between the atom and the radiation field.

Following the procedure we done in the Section 2.2, we can obtain the master equation of the qubit,

$$\begin{aligned} \frac{d\rho^S(t)}{dt} = & -i \frac{\Omega(t)}{2} [\sigma_+ \sigma_-, \rho^S(t)] \\ & + \frac{\gamma(t)}{2} [2\sigma_- \rho^S(t) \sigma_+ - \sigma_+ \sigma_- \rho^S(t) - \rho^S(t) \sigma_+ \sigma_-], \end{aligned} \quad (3.2)$$

where $\Omega(t) = -2\text{Im}[\frac{\dot{c}_0(t)}{c_0(t)}]$, and $\gamma(t) = -2\text{Re}[\frac{\dot{c}_0(t)}{c_0(t)}]$. This is exactly the master equation of the single qubit. $\Omega(t)$ and $\gamma(t)$ play, respectively, the role of time-dependent Lamb shift and decay rate.

With the dynamics of single “qubit + reservoir”, we can readily study the decoherence dynamics of the double-one. We assume, for simplicity, the two systems are the same, and the double-system is initially in a mixed state

$$\rho_{tot}^T(0) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12}^* & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} & \rho_{34} \\ \rho_{14}^* & \rho_{24}^* & \rho_{34}^* & \rho_{44} \end{pmatrix} \otimes |\{0\}_n\rangle \langle \{0\}_n|. \quad (3.3)$$

Following the method we applied in the single qubit system, it is easy to obtain the time evolution of the two qubits. The diagonal elements are

$$\begin{aligned}
\rho_{11}^T(t) &= \rho_{11} |c_0(t)|^4, \\
\rho_{22}^T(t) &= \rho_{22} |c_0(t)|^2 + \rho_{11} |c_0(t)|^2 (1 - |c_0(t)|^2), \\
\rho_{33}^T(t) &= \rho_{33} |c_0(t)|^2 + \rho_{11} |c_0(t)|^2 (1 - |c_0(t)|^2), \\
\rho_{44}^T(t) &= 1 + \rho_{11} |c_0(t)|^4 - |c_0(t)|^2 (2\rho_{11} + \rho_{22} + \rho_{33}),
\end{aligned} \tag{3.4}$$

the nondiagonal elements are

$$\begin{aligned}
\rho_{12}^T(t) &= \rho_{12} |c_0(t)|^2 c_0(t), \\
\rho_{13}^T(t) &= \rho_{13} |c_0(t)|^2 c_0(t), \\
\rho_{14}^T(t) &= \rho_{14} c_0^2(t), \\
\rho_{23}^T(t) &= \rho_{23} |c_0(t)|^2, \\
\rho_{24}^T(t) &= \rho_{24} c_0(t) + \rho_{13} c_0(t) (1 - |c_0(t)|^2), \\
\rho_{34}^T(t) &= \rho_{34} c_0(t) + \rho_{12} c_0(t) (1 - |c_0(t)|^2),
\end{aligned} \tag{3.5}$$

and $\rho_{ij}^T(t) = \rho_{ji}^{T*}(t)$. Differentiating the reduced density matrix with respect to time, we can obtain the equation of motion for the two qubits

$$\begin{aligned}
\frac{d\rho^T(t)}{dt} &= -i \frac{\Omega(t)}{2} ([\sigma_+^A \sigma_-^A, \rho^T(t)] + [\sigma_+^B \sigma_-^B, \rho^T(t)]) \\
&\quad + \frac{\gamma(t)}{2} \{ [2\sigma_-^A \rho^T(t) \sigma_+^A - \sigma_+^A \sigma_-^A \rho^T(t) - \rho^T(t) \sigma_+^A \sigma_-^A] \\
&\quad + [2\sigma_-^B \rho^T(t) \sigma_+^B - \sigma_+^B \sigma_-^B \rho^T(t) - \rho^T(t) \sigma_+^B \sigma_-^B] \},
\end{aligned} \tag{3.6}$$

where $\Omega(t)$ and $\gamma(t)$ are defined the same as before.

To investigate the entanglement dynamics of the bipartite system, we apply Wootters concurrence [67]. The concurrence can be calculated explicitly from the time dependent density matrix $\rho^T(t)$ of the two qubits, $C(\rho^T) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where the quantities λ_i are the eigenvalues of the matrix ζ

$$\zeta = \rho^T(\sigma_y^A \otimes \sigma_y^B) \rho^{T*}(\sigma_y^A \otimes \sigma_y^B), \tag{3.7}$$

arranged in decreasing order. Here ρ^{T*} means the complex conjugation of ρ^T , and σ_y is the Pauli matrix. It can be proved that the concurrence varies from 0 for a separable state to 1 for a maximally entangled state.

As is pointed out, in the Markovian situation the ESD occurs due to the double excitation of the initial state in a vacuum reservoir [15]. In what follows, we consider the initial state of which the concurrence dynamics can exhibit ESD in Markovian approximation, and compare this with the non-Markovian situation.

For an initially entangled pure state in the standard bases

$$\psi(0) = \alpha |--\rangle + \beta |++\rangle, \quad (3.8)$$

α and β satisfy normalization condition. From the time-dependent reduced density matrix of the two qubits and the definition of concurrence, we obtain

$$C(\rho^T) = \max\{0, 2|c_0(t)|^2|\beta| [|\alpha| - |\beta|(1 - |c_0(t)|^2)]\}. \quad (3.9)$$

From the expression of concurrence, it can be found that the behavior of time-dependent factor local excited state population ($|c_0(t)|^2$) completely determines the dynamics of concurrence. In particular, if local system decoherences completely ($|c_0(\infty)|^2 = 0$), entanglement would vanish. Whereas, if decoherence in local system is inhibited, then it is possible to protect entanglement in the long-time limit.

3.3 Mechanism of entanglement preservation

Entanglement is the fundamental resource of quantum information processing. Entanglement preservation is a key step for entanglement applications. To achieve this aim, as we have discussed in Sec. 3.2, one needs the decoherence suppression in the local system. This could be obtained when atom-photon bound state is formed.

Following the same procedure done in Sec. 2.4.3, we obtain the condition for the formation of bound state [59, 66]

$$y(E) \equiv \omega_0 - \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega = E, \quad (3.10)$$

where $J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_k)$ is the spectral density of the reservoir. The solution of Eq. (3.10) highly depends on the explicit form of the $J(\omega)$. If

the reservoir contains only one mode ω' , then $J(\omega) = g^2\delta(\omega - \omega')$. This is the ideal Jaynes-Cummings model [68], in which two bound states in the one excitation sector are formed and as a result the dynamics of the system displays a lossless oscillation. When the reservoir contains infinite modes, one can model $J(\omega)$ by some typical spectrum functions such as the super-Ohmic or Lorentzian form.

We firstly consider the super-Ohmic spectrum $J(\omega) = \eta \frac{\omega^3}{\omega_0^2} e^{-\omega/\omega_c}$, where η is a dimensionless coupling constant and ω_c characterizes the frequency regime in which the power law is valid [61]. It corresponds to that the reservoir consists of a vacuum radiation field, where $g_k \propto \sqrt{\omega_k}$ [68]. The existence of a bound state requires that Eq. (3.10) has at least a real solution for $E < 0$. It is easy to check that the solution always exists if the condition $y(0) < 0$, i.e. $\omega_0 - 2\eta \frac{\omega_c^3}{\omega_0^2} < 0$ is satisfied. Otherwise, no bound state exists. This condition can be fulfilled easily by engineering the environment. For the Lorentzian spectrum it is found that a criterion when a bound state exists can not be obtained analytically. In this case one can use the diagrammatic technique shown later.

The formation of bound state may lead to the inhibition of spontaneous emission and results in the population trapping. Therefore the formation of local bound state has a profound effect on the preservation of non-local coherence(entanglement). One could imagine that non-Markovian effect is also a key factor to preserve entanglement since non-Markovian effect is a kind of backaction effect which could compensate the lost coherence of quantum system. In the following, we explicitly study the formation of bound state and non-Markovian effect on entanglement dynamics using the examples of super-ohmic and Lorentzian vacuum reservoir [66].

Consider firstly the super-Ohmic case. Fig. 3.1 shows the entanglement dynamics in different parameter regimes, i.e., $(\omega_c, \eta) = (0.7\omega_0, 0.2)$, $(0.7\omega_0, 1.0)$ and $(3.0\omega_0, 0.2)$. For the first two parameter sets the bound state is absent, while for the last one it is available, as shown in Fig. 3.1 (a). Whether the bound state exists or not plays a key role in the entanglement preservation in the long time limit. When the bound state is absent, the residual entanglement approaches zero in a long enough time, as shown by the solid lines in

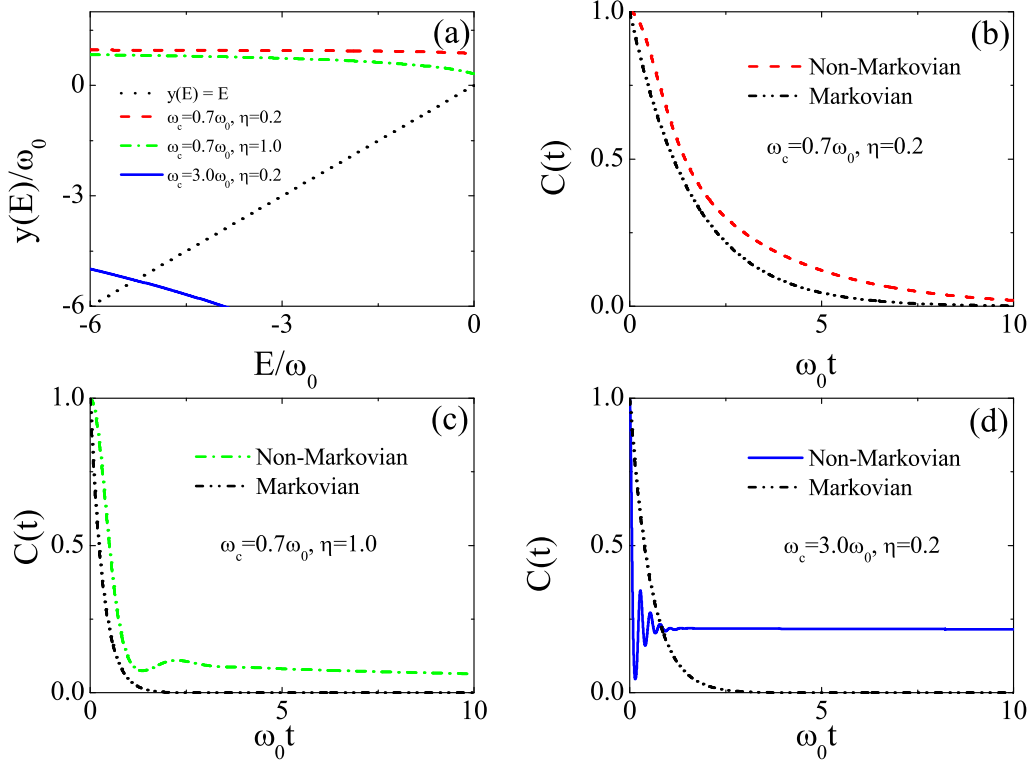


Figure 3.1: Entanglement dynamics of the two-qubit system with local super-Ohmic reservoirs. (a) Diagrammatic solutions of Eq. (3.10) with different parameters. $C(t)$ as a function of time is shown in (b): $(\omega_c, \eta) = (0.7\omega_0, 0.2)$, (c): $(\omega_c, \eta) = (0.7\omega_0, 1.0)$ and (d): $(\omega_c, \eta) = (3.0\omega_0, 0.2)$. The parameter α is taken as 0.7. For comparison, $C(t)$ under the Markovian approximation has also been presented by using the same parameters.

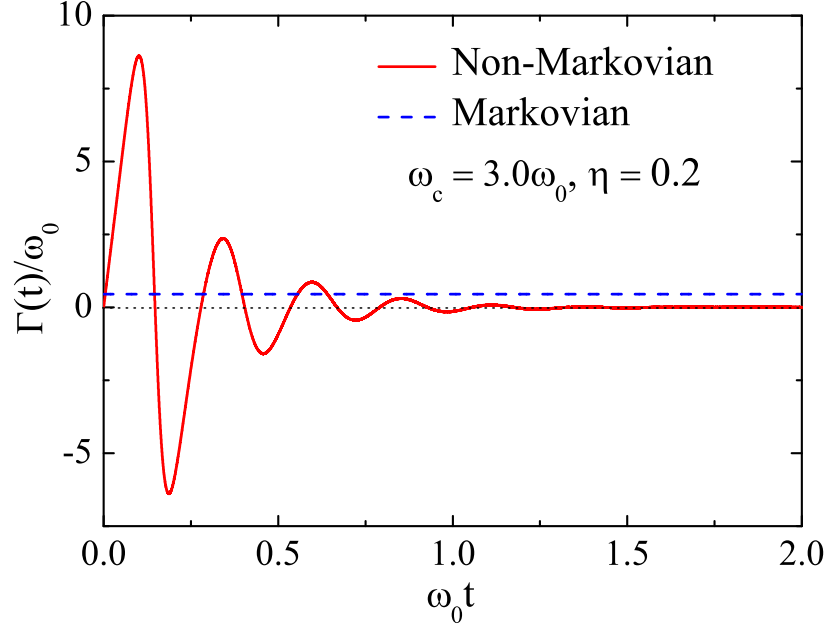


Figure 3.2: The decay rate $\Gamma(t)$ as a function of time in the non-Markovian and Markovian cases. The parameters used are $\omega_c = 3.0\omega_0$ and $\eta = 0.2$.

Fig. 3.1 (b) and (c). Difference between these two cases is that Fig. 3.1(b) is in weak coupling regime, where the non-Markovian effect is weak, while Fig. 3.1(c) is in strong coupling regime, where the strong non-Markovian effect leads to an obvious oscillation. When the bound state is available, the situation is quite different, as shown in Fig. 3.1(d). The entanglement firstly experiences some oscillations due to the energy and/or information exchanging back and forth between the qubit and its memory environment [48], then approaches a definite value in the long time limit, where the decay rate approaches zero after some oscillations, as shown in Fig. 3.2. *The entanglement preservation is a result from the interplay between the existence of the bound state (providing an ability to preserve the entanglement) and the non-Markovian effect (providing a way to preserve the entanglement)* [66]. The claim can be further verified by that the entanglement preservation is absent in the Markovian dynamics, as shown by the dashed lines in Fig. 3.1 (b), (c) and (d), where the entanglement displays sudden death irrespective

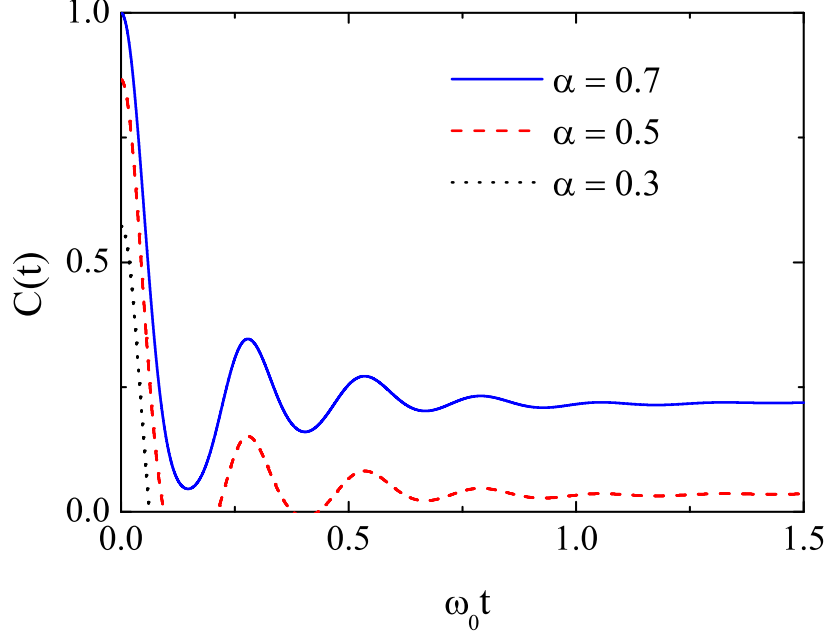


Figure 3.3: The residual entanglement for different initial states with $\alpha = 0.7, 0.5$ and 0.3 . The other parameters used are the same as those in Fig. 3.1.

of the availability of the bound state. It is because the Markovian environment has no memory and the energy/information flowing from the qubit to its environment is irreversible and the decay rate keeps to be fixed (see, Fig. 3.2). In this case one has

$$C(t) = \max\{0, 2e^{-2\Gamma_0 t} |\beta| [|\alpha| - |\beta| (1 - e^{-2\Gamma_0 t})]\}, \quad (3.11)$$

which shows a finite disentanglement time when $|\alpha| < |\beta|$ [14]. In a word, the above discussion manifests clearly two conditions to preserve the entanglement, i.e., the availability of the bound state and the non-Markovian effect [66], not only the structured environment as emphasized in Ref. [28].

The above discussion focused on almost maximally entangled initial state by taking $\alpha = 0.7$. In Fig. 3.3 we show the results for different initial states with different initial entanglement. With decreasing the initial entanglement, the residual entanglement also decreases in the long time limit and finally, the ESD happens for $\alpha = 0.3$. The result can be understood from Eq. (3.9).

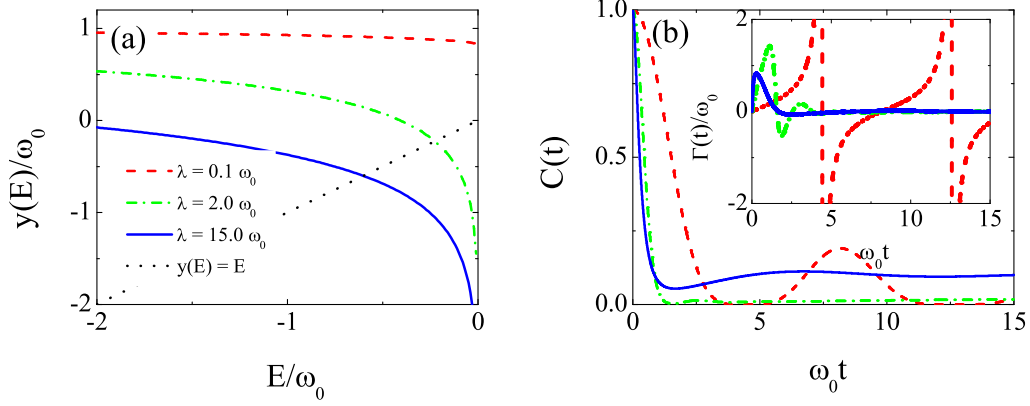


Figure 3.4: The entanglement dynamics with the Lorentzian spectrum. (a) Diagrammatic solutions of Eq. (3.10) with different parameters $\lambda = 0.1\omega_0, 2.0\omega_0$ and $15\omega_0$. (b) $C(t)$ as a function of time for the corresponding three parameter regimes. The insert in (b) are the decay rate as a function of time. The other parameters used are $\gamma = 3.0\omega_0$ and $\alpha = 0.7$.

On the one hand, the residual entanglement is determined by $c_0(\infty)$, which is directly related to the property of the bound state. On the other hand, the residual entanglement is also determined by the competition between the first and the second terms in Eq. (3.9), which is dependent of the initial state.

In order to make a comparative study and confirm our observations we consider the Lorentzian spectrum if the reservoir is composed of lossy cavity,

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma\lambda^2}{(\omega - \omega_0)^2 + \lambda^2}, \quad (3.12)$$

where γ is the coupling constant and λ is the spectrum width. This model has also been studied in Ref. [29], where the lower limit of frequency integral in $f(t - \tau)$ was extended from zero to negative infinity. This extension is mathematically convenient but the availability of the bound state is missed. Here we follow the original definition of the frequency integral ranges.

Our model with the Lorentzian spectral density corresponds exactly to the extended damping J-C model [1]. It is noted that the strong coupling of J-C model has been achieved in circuit QED [69] and quantum dots [70] systems. Fig. 3.4 shows the entanglement dynamics of the qubits under

the Lorentzian reservoir for different spectrum widths in the strong coupling regime. When $\lambda = 0.1\omega_0$, Eq. (3.10) lacks the bound state. According to the above discussion, there is no residual entanglement in the long time limit. This is indeed true, as shown in Fig. 3.4 (b). However, it is noted that before becoming zero the entanglement exhibits “sudden death” and revives after some time for several times. This is an analog of the central result found in Ref. [29], i.e., the phenomenon of the ESD and revival. Apparently, this is due to the non-Markovian effect, the revival is a result of backaction of the memory reservoir. When increasing the spectrum width the bound states become available, the situation changes. The significant fraction of the entanglement initially present is preserved in the long time limit, where the decay rates shown in the insert of Fig. 3.4 (b) approach zero in these cases. Likewise, the physical nature of the entanglement preservation is still the interplay between the bound state and the non-Markovian effect. The more stronger the coupling is, the more striking the entanglement oscillates as a function of time, consequently, the more noticeable the non-Markovian effect is, as shown in Fig. 3.5. For $\gamma = 0.2\omega_0$, the system is in the weak coupling regime, where the bound state is also not available. As a result, the ESD is reproduced in this case.

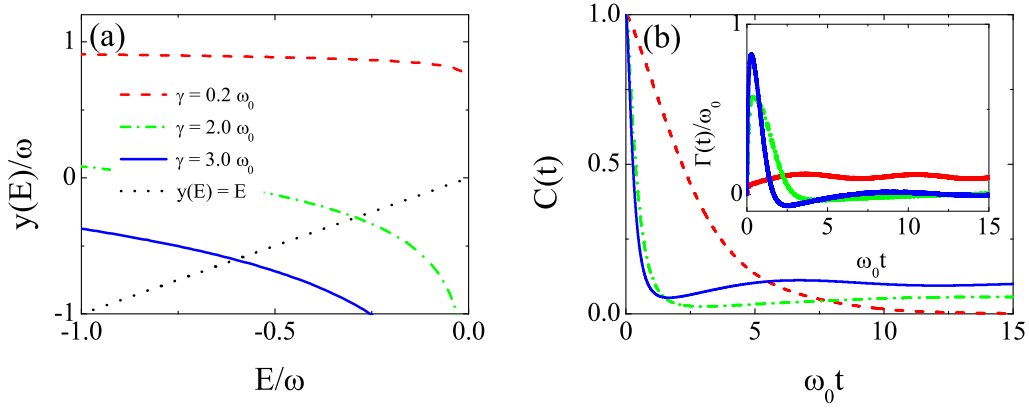


Figure 3.5: The same as Fig. 3.4 but $\lambda = 15.0\omega_0$ is fixed and $\gamma = 0.2\omega_0, 2.0\omega_0$ and $3.0\omega_0$.

In Fig. 3.6 we present a phase diagram of the entanglement in the steady state for the Lorentzian spectral density. In the large γ and small λ regime,

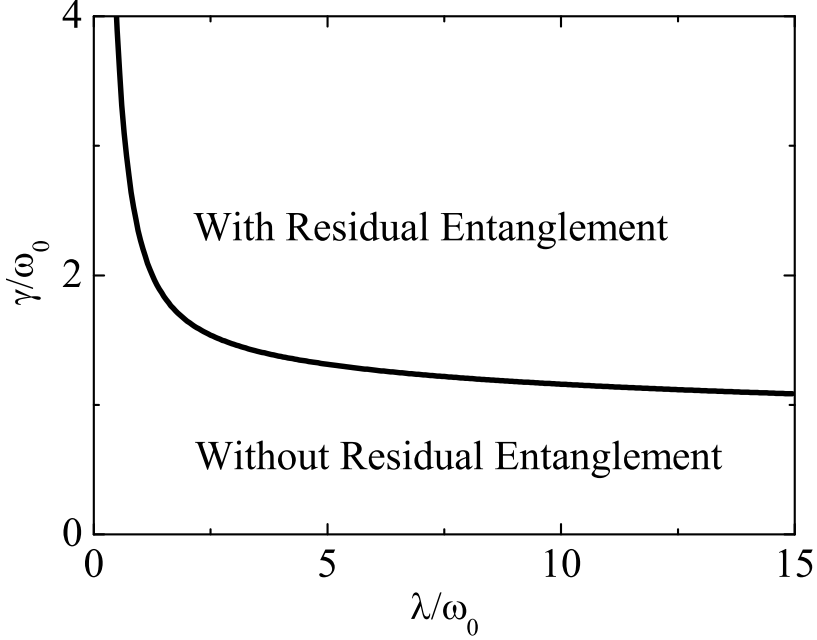


Figure 3.6: The phase diagram of the residual entanglement in the steady state for Lorentzian spectrum.

the system approaches the J-C model. In this situation the strong backaction effect of the reservoirs makes the qubit system hard to form a steady state. The entanglement oscillates with time but has no dissipation. In the small γ and large λ regime, the non-Markovian effect is extremely weak and our results reduce to the Markovian case. In a limit of the flat spectral density, the Born-Markovian approximation is applicable and the system has no bound state. This is the case of the ESD [14, 66].

3.4 Summary

In this chapter, we have studied the entanglement protection of two qubits in two uncorrelated reservoirs. Two essential conditions to preserve the entanglement are explored, one is the existence of the bound state of the system and its reservoir and the other one is the non-Markovian effect. The bound state provides the ability of the entanglement preservation and the

non-Markovian effect provides the way to protect the entanglement. The previous results on the entanglement dynamics in the literature can be considered as the specific cases where these two conditions have not been fulfilled at the same time. The result provides a unified picture for the entanglement dynamics and gives a clear way how to protect the entanglement. This is quite significant in the quantum information processing.

The presence of such entanglement preservation gives us an active way to suppress decoherence. This could be achieved by modifying the spectrum of the reservoirs to approach the non-Markovian regime and form a bound state via the potential usage of the reservoir engineering technique [55, 71, 72]. Fortunately, we notice that many practical systems have now be engineered to show strong non-Markovian effect [19, 20, 21, 22]. All these achievements have paved the way to experimentally simulate the paradigmatic models of open quantum system, which gives a hopeful prospective to preserve the entanglement.

Chapter 4

Entanglement distribution and its invariance

In this chapter we study the entanglement distribution among bipartite systems of quantum systems and their reservoirs. Following the method used in Ref. [40], we find an invariant and entanglement can be distributed among all the bipartite subsystems.

4.1 Introduction

In Chapter 3, we have study the entanglement dynamics under different environments. We have figured out the mechanism of entanglement preservation, i.e. entanglement can be preserved when bound states are formed under the non-Markovian dynamics.

Recently, López *et al.* asked a question about where the lost entanglement between the qubits goes [40]. Interestingly, they found that the lost entanglement of the qubits is exclusively transferred to the reservoirs under the Markovian dynamics and the ESD of the qubits is always accompanied with the entanglement sudden birth (ESB) of the reservoirs. This means that the entanglement does not go away, it is still there but just changes the location. This is reminiscent of the work of Yonac *et al.* [39], in which the entanglement dynamics has been studied in a double J-C model. They found that the entanglement is transferred periodically among all the bipartite partitions of the whole system but an identity (see below) has been satisfied at any time.

This may be not surprising since the double J-C model has no decoherence and any initial information can be preserved in the time evolution. However, it would be surprising if the identity is still valid in the presence of the decoherence, in which a non-equilibrium relaxation process is involved. In this chapter, we show that it is indeed true for such a system consisted of two qubits locally interacting with two reservoirs. We find that the distribution of the entanglement among the bipartite subsystems is dependent of the explicit property of the environment and its coupling with the qubit. The rich dynamical behaviors obtained previously in the literature can be regarded as the special cases of our present result or Markovian approximation. Particularly, we find that the entanglement can stably distribute among all the bipartite subsystems if the qubit and its environment can form a bound state and the non-Markovian effect is important. Irrespective of how distributes the entanglement, it is found that the identity about the entanglement in the whole system can be satisfied at any time, which reveals the profound physics of the entanglement dynamics.

4.2 The model of two qubits in two uncorrelated band-gap reservoirs

The model we studied here is the same as the one used in chapter 3, i.e. two qubits in two separate vacuum reservoirs. Because of the independence of the two subsystems, we first consider the single “qubit + reservoir” subsystem which is governed by the following Hamiltonian [68]

$$H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (g_k \sigma_+ a_k + h.c.), \quad (4.1)$$

in which the notations are the same as Sec. 2.2

As we have discussed in chapter 2, the initial state of the local system $|\phi(0)\rangle = |-\rangle \otimes |\{0\}_k\rangle$, where $|\{0\}_k\rangle$ denotes the vacuum state of reservoir, does not evolve with time. While for an initial state $|\phi(0)\rangle = |+\rangle \otimes |\{0\}_k\rangle$, governed by the Hamiltonian (4.1), its time evolution is given by

$$|\phi(t)\rangle = b(t) |+, \{0\}_k\rangle + \sum_k b_k(t) |-, \{1\}_k\rangle, \quad (4.2)$$

where $|\{1\}_k\rangle$ denotes the reservoir state with only one photon in the k -th mode. From the Schrödinger equation, we can get the time evolution of the excited probability amplitudes in Eq. (4.2)

$$\dot{b}(t) + i\omega_0 b(t) + \int_0^t b(\tau) f(t - \tau) d\tau = 0, \quad (4.3)$$

where the kernel function is $f(x) = \int_0^\infty d\omega J(\omega) e^{-i\omega x}$ with $J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_k)$ being the spectral density.

If we define the normalized collective state with one excitation in the reservoir as $|\tilde{\mathbf{1}}\rangle_r = [\sum_k b_k(t) |\{1\}_k\rangle] / \tilde{b}(t)$ and with no excitation in the reservoir as $|\tilde{\mathbf{0}}\rangle_r = |\{0\}_k\rangle$, then Eq. (4.2) can be recast into [40, 73]

$$|\phi(t)\rangle = b(t) |+\rangle |\tilde{\mathbf{0}}\rangle_r + \tilde{b}(t) |-\rangle |\tilde{\mathbf{1}}\rangle_r, \quad (4.4)$$

where $\tilde{b}(t) = \sqrt{1 - |b(t)|^2}$.

According to the above results, the time evolution of a system consisted of two such subsystems with the initial state $|\Phi(0)\rangle = (\alpha |-, -\rangle + \beta |+, +\rangle) |\tilde{\mathbf{0}}\rangle_{r_1} |\tilde{\mathbf{0}}\rangle_{r_2}$ is given by

$$|\Phi(t)\rangle = \alpha |-, -\rangle |\tilde{\mathbf{0}}\rangle_{r_1} |\tilde{\mathbf{0}}\rangle_{r_2} + \beta |\phi(t)\rangle_1 |\phi(t)\rangle_2, \quad (4.5)$$

where α and β are the coefficients to determine the initial entanglement in the system. From $\rho = |\Phi(t)\rangle \langle \Phi(t)|$, one can obtain the time-dependent reduced density matrix of the bipartite subsystem qubit1-qubit2 ($q_1 q_2$) by tracing over the reservoir variables. It reads

$$\rho_{q_1 q_2}(t) = \begin{pmatrix} |\beta|^2 |b(t)|^4 & 0 & 0 & \beta \alpha^* b(t)^2 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ \beta^* \alpha b^*(t)^2 & 0 & 0 & x \end{pmatrix}, \quad (4.6)$$

where $p = |\beta b(t)|^2 \tilde{b}(t)^2$ and $x = 1 - |\beta|^2 |b(t)|^4 - 2p$. By the similar procedure, it is not difficult to obtain the corresponding reduced density matrices for other bipartite subsystems like reservoir1-reservoir2 ($r_1 r_2$) and qubit-reservoir ($q_1 r_1, q_1 r_2, q_2 r_1, q_2 r_2$).

4.3 Entanglement distribution among bipartite subsystems

Follow chapter 3, we use the concurrence [67] to quantify entanglement. The concurrence for each bipartite partition can be calculated as $C_m = \max\{0, Q_m\}$, where m denotes the different bipartite partitions and Q_m read [73]

$$Q_{q_1 q_2} = 2|\alpha\beta||b(t)|^2 - 2|\beta b(t)|^2 \tilde{b}(t)^2, \quad (4.7)$$

$$Q_{r_1 r_2} = 2|\alpha\beta|\tilde{b}(t)^2 - 2|\beta b(t)|^2 \tilde{b}(t)^2, \quad (4.8)$$

$$Q_{q_i r_i} = 2|\beta|^2 |b(t)|\tilde{b}(t) \quad (i = 1, 2), \quad (4.9)$$

$$Q_{q_1 r_2} = 2|\alpha\beta b(t)|\tilde{b}(t) - 2|\beta b(t)|^2 \tilde{b}(t)^2 = Q_{q_2 r_1}. \quad (4.10)$$

It is straightforward to verify that the quantities Q_m in Eqs. (4.7)-(4.10) satisfy an identity

$$Q_{q_1 q_2} + Q_{r_1 r_2} + 2 \left| \frac{\alpha}{\beta} \right| Q_{q_1 r_1} - 2Q_{q_1 r_2} = 2|\alpha\beta|, \quad (4.11)$$

where $2|\alpha\beta|$ is just the initial entanglement. Eq. (4.11) has been obtained in a double J-C model [39], in which the decoherence is absent since each of the reservoirs only contains one mode, i.e. $J(\omega) = g^2 \delta(\omega - \omega_0)$. Surprisingly, this identity is still true even in the presence of decoherence. Furthermore, one notes that the identity is not dependent of any detail about $b(t)$, which is determined by Eq. (4.3). This result shows clearly the invariant nature of the entanglement. In the following we explicitly discuss the distribution behavior of the entanglement by taking the reservoir as a photonic band gap (PBG) medium [25, 37] and compare it with the previous results.

For the PBG medium, the dispersion relation near the upper band-edge is given by [26]

$$\omega_k = \omega_c + A(k - k_0)^2, \quad (4.12)$$

where $A \approx \omega_c/k_0^2$, ω_c is the upper band-edge frequency and k_0 is the corresponding characteristic wave vector. In this case, the kernel function has the form [73]

$$f(t - \tau) = \eta \int \frac{c^3 k^2}{\omega_k} e^{-i\omega_k(t-\tau)} dk, \quad (4.13)$$

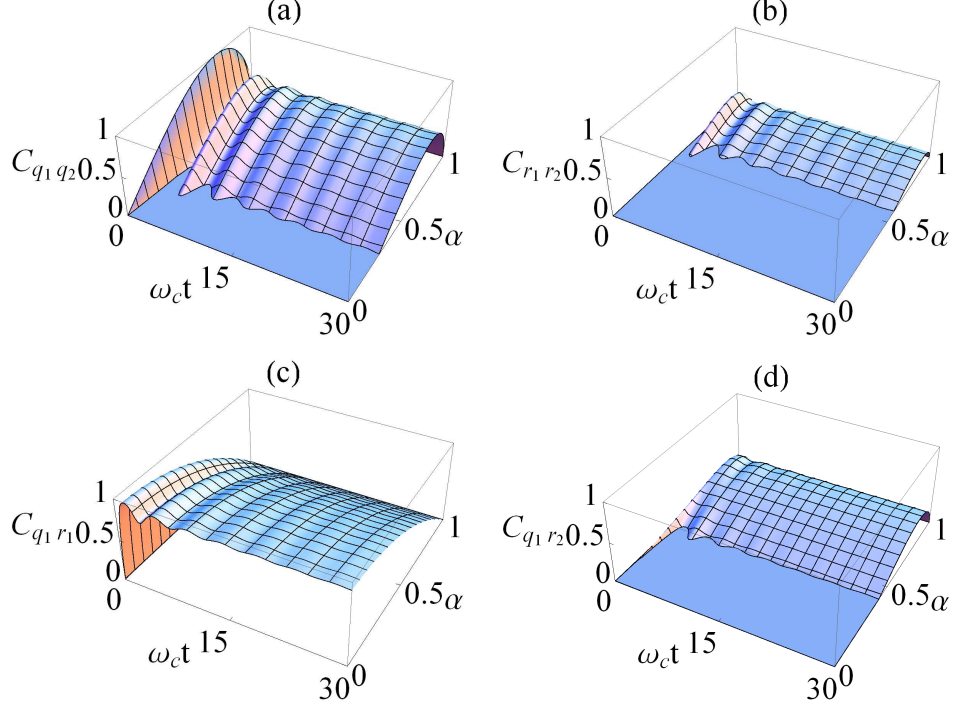


Figure 4.1: Entanglement distributions and their time evolutions for the case of $\omega_0 < \omega_c$. The parameters used are $\omega_0 = 0.1\omega_c$ and $\eta = 0.2$.

where $\eta = \frac{\omega_0^2 d^2}{6\pi^2 \epsilon_0 c^3}$ is a dimensionless constant. In solving Eq. (4.3) for $b(t)$, Eq. (4.13) is evaluated numerically. Here we do not make an assumption that k can be replaced by k_0 outside of the exponential [36], as also done in Refs. [28, 74, 75]. Thus our result is numerically exact. In the following discussion we take ω_c as the unit of frequency.

In Figs. 4.1 and 4.2, we show the entanglement distributions and their time evolutions for two typical cases of $\omega_0 < \omega_c$ and $\omega_0 > \omega_c$, which correspond to the atomic frequency being located at the band gap and at the upper band of the PBG medium, respectively [73]. In the both cases the initial entanglement in $q_1 q_2$ begins to transfer to other bipartite partitions with time but their explicit evolutions, in particular the long time behaviors, are quite different. In the former case, the entanglement could distribute stably among all bipartite partitions. Fig. 4.1(a) shows that after some

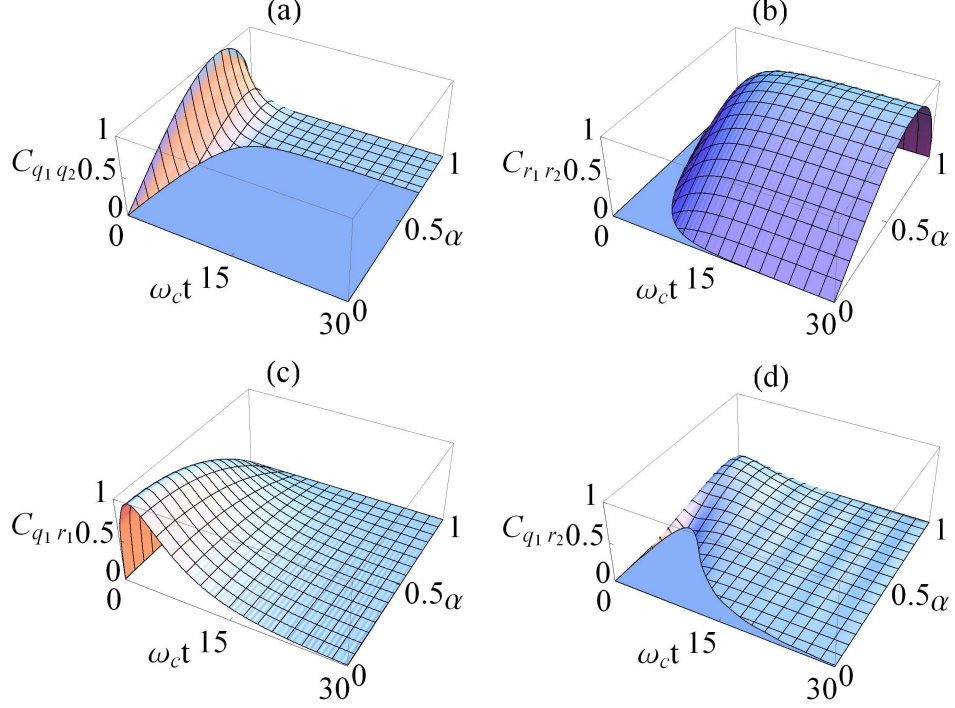


Figure 4.2: Entanglement distributions and their time evolutions for the case of $\omega_0 > \omega_c$. The parameters used are $\omega_0 = 10.0\omega_c$ and $\eta = 0.2$.

oscillations, a sizeable entanglement of $q_1 q_2$ is preserved for the parameter regime of $0.3 < \alpha < 1$ in the long-time limit. Remarkably, the entanglement in $q_i r_i (i = 1, 2)$ forms quickly in the full range of α [Fig. 4.1(c)] and dominates the distribution. On the contrary, only slight entanglement of $r_1 r_2$ is formed in a very narrow parameter regime $0.6 < \alpha < 1$, as shown in Fig. 4.1(b). However, when ω_0 is located at the upper band of the PBG medium, the initial entanglement in $q_1 q_2$ is transferred completely to the $r_1 r_2$ in the long-time limit, as shown in Fig. 4.2. At the initial stage, $q_i r_i (i = 1, 2)$ and $q_1 r_2 (q_2 r_1)$ are entangled transiently, but there is no stable entanglement distribution. This result is consistent with that in Refs. [40, 76, 77]. It is not difficult to understand these results according to Eqs. (4.7)-(4.10). From these equations, one can clearly see that the detailed behavior of the entanglement dynamics and its distributions in the bipartite partitions are

completely determined by the time-dependent factor $|b(t)|^2$ of single-qubit excited-state population. Fig. 4.3 shows its time evolutions for the corresponding parameter regimes presented above. We notice that $|b(\infty)|^2 \neq 0$ when ω_0 is located at the band gap, which means that there is some excited-state population in the long-time limit. This is just the population trapping which we have discussed in above chapters. Such population trapping just manifests the formation of bound states between q_i and r_i [66]. Consequently, q_i and r_i are so correlated in the bound states that the initial entanglement in q_1q_2 cannot be fully transferred to r_1r_2 . The oscillation during the evolution is just the manifestation of the strong non-Markovian effect induced by the reservoirs. On the contrary, if ω_0 is located in the upper band, then $|b(\infty)|^2 = 0$ and the qubits decay completely to their ground states. In this case the bound states between q_i and r_i are absent and the initial entanglement in q_1q_2 is completely transferred to the r_1r_2 , as clearly shown in Eq. (4.8).

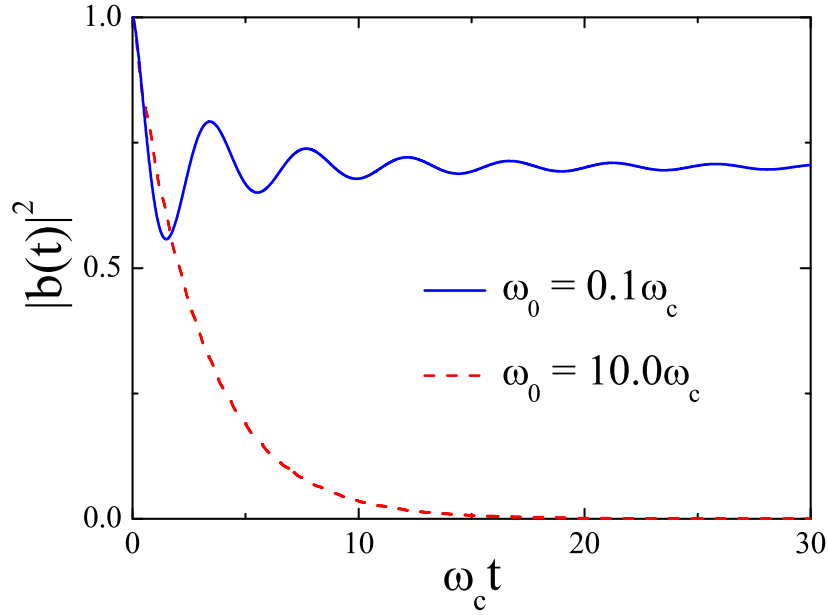


Figure 4.3: Time evolution of time-dependent factor of the excited-state population for two parameter regimes $\omega_0 = 0.1\omega_c$ (solid line) and $10.0\omega_c$ (dashed line). η is taken as 0.2.

In addition, in Refs. [40, 76, 77] it was emphasized that the ESD of q_1q_2 is always accompanied with the ESB of r_1r_2 . However, this is not always true. To clarify this, we examine the condition to obtain the ESD of the qubits and the accompanying ESB of the reservoirs. From Eqs. (4.7) and (4.8) it is obvious that the condition is $Q_{q_1q_2}(t) < 0$ and $Q_{r_1r_2}(t') > 0$ at any t and t' , which means [73]

$$|b(t')|^2 < |\alpha|/\sqrt{1-|\alpha|^2} < 1 - |b(t)|^2. \quad (4.14)$$

In the case without bound states, $|b(\infty)|^2 = 0$. The condition (4.14) can be satisfied when $\alpha < 1/\sqrt{2}$. So one can always expect the ESD of the qubits and the accompanying ESB of the reservoirs in the region $|\alpha| < 1/\sqrt{2}$, as shown in Fig. 4.2 and Refs. [40, 76, 77]. However, when the bound states are available, the situation changes. In particular, when $|b(t)|^2 \geq \frac{1}{2}$ in the full range of time evolution, no region of α can make the condition (4.14) to be satisfied anymore. For clarification, we present three typical behaviors of the entanglement distribution in Fig. 4.4. In all these cases the bound states are available. Fig. 4.4 (a) shows the situation where the entanglement is stably distributed among all of the bipartite subsystems. In Fig. 4.4 (b) the entanglement of r_1r_2 shows ESB and revival. However, the entanglement in q_1q_2 does not exhibit ESD. This is the example that the ESD in q_1q_2 is not accompanied with the ESB in r_1r_2 . Fig. 4.4(c) shows another example that while the entanglement of q_1q_2 shows ESD and revival [29], the entanglement of r_1r_2 does not show ESB but remains to be zero.

The above discussion is general and is not dependent of the explicit form of the reservoir. To confirm this, we consider the radiation field in free space. The spectral density has the Ohmic form $J(\omega) = \eta\omega \exp(-\omega/\Lambda)$, which can be obtained from the free-space dispersion relation $\omega = ck$. One can verify that the condition for the formation of bound states is: $\omega_0 - \eta\Lambda < 0$ [66]. In Fig. 4.5, we plot the results in this situation. The previous results can be recovered when the bound states are absent. On the contrary, when the bound states are available, a stable entanglement is established among all the bipartite partitions. Therefore, we argue that the stable entanglement distribution resulted from the bound states is a general phenomenon in open

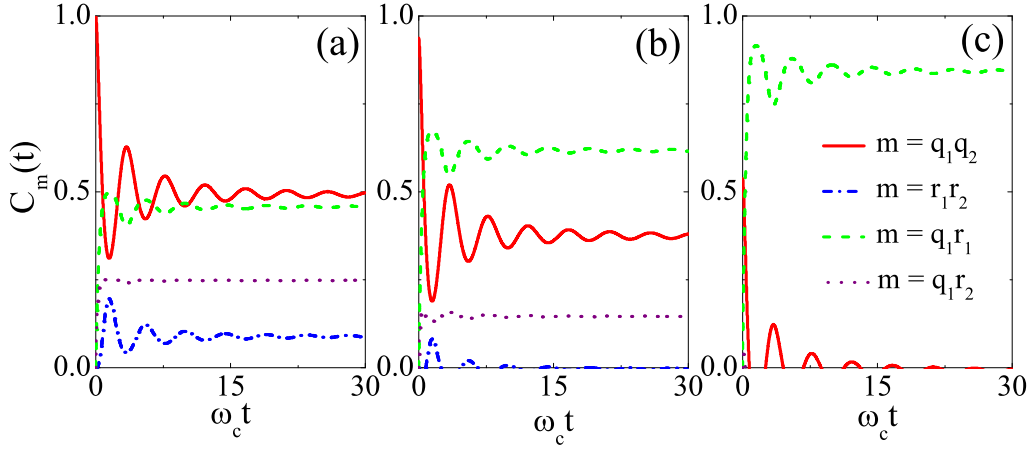


Figure 4.4: (Color online) Entanglement evolution when $\alpha = 1/\sqrt{2}$ (a), $\alpha = 0.57$ (b), and $\alpha = 0.28$ (c). The parameters used here are the same as Fig. 4.1.

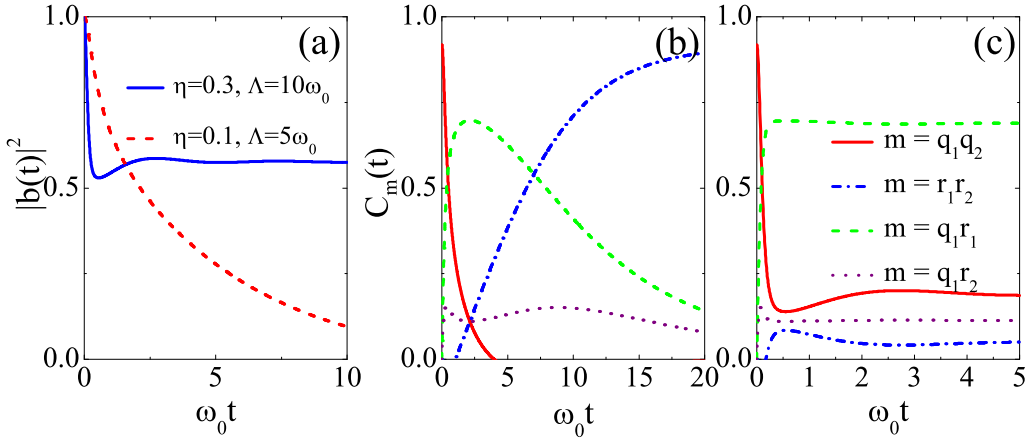


Figure 4.5: (Color online) Entanglement evolution for the Ohmic spectral density. The two sets of parameters $(\eta, \Lambda) = (0.1, 5\omega_0)$ and $(0.3, 10\omega_0)$ have been considered for comparison. The corresponding entanglement distributions and their evolution are given in (b) and (c), respectively. In both cases $\alpha = 0.55$.

quantum system when the non-Markovian effect is taken into account.

4.4 Summary

In summary, we have studied the entanglement distribution among all the bipartite subsystems of two qubits embedded into two independent reservoirs. It is found that the entanglement can be stably distributed in all the bipartite subsystems and they satisfy an identity about the entanglement. This identity is shown to be independent of any detail of the reservoir and its coupling with the qubit, which affect only the explicit time evolution behavior and the final distribution. The result shows the physical nature of the entanglement and has a significant implication for the quantum information processing.

Chapter 5

Summary and outlook

In this thesis we studied the decoherence dynamics of open quantum system. We found that the decoherence would be greatly suppressed if the bound state is formed under the non-Markovian dynamics.

We model our system as two-level atoms in vacuum reservoirs. After numerically solving the coupled equations, we studied the non-Markovian effect on the decoherence dynamics of the quantum system. Compared with the results obtained under the Markovian approximation, we found that the environment has two effects on the quantum system of interest: dissipation effect, which degrades the quantum coherence, and backaction effect, which compensates the quantum system of lost coherence. The competition of these two effects results in the rich decoherence dynamic behaviors. Our results show explicitly, in the weak coupling regime, the widely used Markovian approximation is applicable, while, in the strong coupling regime, the non-Markovian effect makes the dynamical process oscillate for a certain time which is a manifestation of information or energy flowing back and forth between quantum system and the environment. We also find that the quantum coherence can be preserved in the long time limit. Physically, it is attribute to the formation of a bound state between the quantum system and its local reservoir. We give explicitly a condition to judge the formation of bound state for any kind of vacuum reservoir including the widely studied PBG medium.

Due to the preservation of quantum coherence of the single system, we have revealed that the entanglement for a composite two-qubit system can

also be preserved in the steady state. We give explicitly the mechanism of entanglement preservation, i.e. the fulfillment of non-Markovian effect and formation of bound states. The mechanism we given can explain the results obtained in the previous works. When the non-Markovian effect is neglected, the phenomenon of ESD of the qubits is reproduced. The phenomenon of ESD and its revival can be obtained when the non-Markovian effect is contained while the bound state is not available. In particular, the entanglement preservation when the atoms are placed in the PBG mediums reported in Ref. [28] can be explained as the fulfillment of the above two conditions. In a word, we have given a clear clue on how to preserve entanglement in the steady state.

Considering the environment as a whole, we also investigated the entanglement distribution among all the bipartite subsystems. We found that the entanglement can be stably distributed among all bipartite partitions of the whole system when the bound states are formed. It is particularly interesting to find that the entanglement in different bipartite partitions always satisfies an identity. This identity is independent on the explicit dynamics process. Our unified treatment includes the previous results in the literature as special cases. When the bound state is absent and the Markovian approximation is applicable, the result reported in Ref. [40] that the entanglement transfer from the qubits to the reservoirs is recovered. Our work give a thorough understanding of entanglement distribution among quantum systems and their environments.

There are many open issues relevant to the subject of this thesis. For example, the mechanism of formation of bound state is still unclear, we think it is a kind of quantum phase transition when the bound state is formed. How to reveal the relationship between the bound state and the quantum phase transition is an open question. What's more, in this thesis we have shown that the non-Markovian effect can rescue the entanglement. Does the non-Markovian effect also can rescue certain missions of quantum information processing, for example, quantum teleportation and quantum dense coding, in noisy quantum channels? This is still an open question.

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Publication list

1. **Qing-Jun Tong**, Jun-Hong An, Hong-Gong Luo, and C. H. Oh, *Mechanism of entanglement preservation*, Phys. Rev. A **81**, 052330 (2010).
2. Juan-Juan Chen, Jun-Hong An, **Qing-Jun Tong**, Hong-Gang Luo, and C. H. Oh, *Non-Markovian effect on the geometric phase of a dissipative qubit*, Phys. Rev. A **81**, 022120 (2010).
3. **Qing-Jun Tong**, Jun-Hong An, Hong-Gong Luo, and C. H. Oh, *Decoherence suppression of a dissipative qubit by non-Markovian effect*, J. Phys. B: At. Mol. Opt. Phys. **43** 155501 (2010).
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